Introduction: Mensuration is the part of mathematics that deals in measurement of lengths, areas and volumes of regular as well as irregular figures. In this chapter we are going to discuss about the area and volume of some specific two-dimensional and three-dimensional figures.

3.1. Area of Two-Dimensional Figures: Polygons.

A Polygon is a closed figure formed by line segments such that no two line segments intersect or coincides with each other. *A Polygon is called a regular polygon if all of its sides are equal in length*. Polygons are classified according to the number of sides or vertices as follows:

No. of sides or vertices	Name of the Polygon
Three	Triangle
Four	Quadrilateral
Five	Pentagon
Six	Hexagon
Seven	Heptagon
Eight	Octagon
Nine	Nonagon
Ten	Decagon

Area of a regular polygon of n-sides:

ABCDEF is a regular polygon of six sides. We draw two circles, one circumscribed about it and another inscribed in it. O is their common centre.R is the radius of the circumscribed circle and r is the radius of the inscribed circle. The polygon can be divided into six equal triangles. *The six sided regular polygon can be extended to an n-sided regular polygon and we can conclude that,*

(i) the polygon can be divided into nnumber of equal triangles,
(ii) the line joining the central point to any of the vertices of the polygon is R,
(iii) the line drawn perpendicular from the central point to any of the sides is r,
(iv) the perimeter of the polygon is n side length.

(a) Area of a n-sided regular polygon of side a:

The polygon is divided into n equal triangles, OAB is one of them.
Area of the polygon = n Area of OAB
= n AB OG

Area of a regular polygon of n-sides:

ABCDEF is a regular polygon of six sides. We draw two circles, one circumscribed about it and another inscribed in it. O is their common centre.R is the radius of the circumscribed circle and r is the radius of the inscribed circle. The polygon can be divided into six equal triangles.

i
) the polygon can be divided into nnumber of equal triangles,

(ii) the line joining the central point to any of the vertices of the polygon is R,
(iii) the line drawn perpendicular from the central point to any of the sides is r,
(iv) the perimeter of the polygon is n side length.

Next, we will obtain the area of a regular polygon of n-sides under different conditions:

(a) Area of a n-sided regular polygon of side a:

= ar

(b) Area of a regular polygon of n-sides with side a and R given:

=

(c) Area of a regular polygon of n-sides with side a:

Required Area $=a^2$ Cot

(d) Area of a regular polygon of n-sides with r given:

RECTILINEAL FIGURES AND CURVILINEAL FIGURES:

Required Area = n^2tan

We have discussed about regular rectilineal figures in the last section. In this section we will discuss about irregular rectilinear figures and curvilineal figures.

3.2.1 Irregular Rectilineal Figures:

An irregular rectilineal figure is first divided into different parts, the area of each of these parts are obtained, the sum of all these areas will be the required area.

RECTILINEAL FIGURES AND CURVILINEAL FIGURES:

We have discussed about regular rectilineal figures in the last section. In this section we will discuss about irregular rectilinear figures and curvilineal figures.

3.2.1 Irregular Rectilineal Figures:

An irregular rectilineal figure is first divided into different parts, the area of each of these parts are obtained, the sum of all these areas will be the required area.

3.2.2 Curvilineal Figures:

A curvilineal figure is a closed figure, either completely bounded by a closed curve as shown in figure (i) or bounded by a curve on one side only as shown in figure (ii).

figure (i) figure (ii)

Area of a curvilineal figure can be obtained using a rule known as SIMPSONS ONE THIRD RULE. Now we proceed to observe how Simpsons rule works for different curvilineal figures.

by SIMPSONS ONE THIRD RULE as follows: According to this rule,

Required Area = [(first ordinate + last ordinate) +2(sum of the remaining odd ordinates) + 4(sum of even ordinates)]

$= [\ (y_1+y_{2n+1}) + 2(\ y_3+y_5+y_7+..+y_{2n-1}) + 4(\ y_2+y_4+y_6+..+y_{2n})]$

Ex 3 (2009) : Find by Simpsons rule the area of the curvilinear figure whose ordinates measure 18, 22, 26, 24, 20, 26, 30, 34, 28, 24, 14 metres and whose base is 150 metres.

Solution : Given ordinates are(in ms):

$y_1 = 18$, $y_2 = 22$, $y_3 = 26$, $y_4 = 24$, $y_5 = 20$, $y_6 = 26$, $y_7 = 30$, $y_8 = 34$, $y_9 = 28$, $y_{10} = 24$, $y_{11} = 14$;

.

The base length = 150 ms, number of ordinates = 11, number of divisions = 10 and d = m = 15 m.

By Simpsons rule, required area

$= [(y_1+y_{11}) + 2(y_3+y_5+y_7+y_9) + 4(y_2+y_4+y_6+y_8+y_{10})]$

$= [(18+14) + 2(26+20+30+28) + 4(22+24+26+34+24)]$ sq. m

= 5 [32 + 2104 + 4 x 130] sq. m = 3800 sq. m

Example 9 (2013) : The cross sectional area of a tunnel is as follows:

Distance from one end :	0	3	6	9	12	15	18
Areas :	27.9	30.6	33.8	32.4	30.7	27.9	26.1

Find the volume of the solid.

Solution: Areas are considered as ordinates,

$y_1=27.9$, $y_2=30.6$, $y_3=33.8$, $y_4=32.4$, $y_5=30.7$ $y_6=27.9$, $y_7=26.1$; d=3

According to Simpsons one third rule,

Example 9 (2013) : The cross sectional area of a tunnel is as follows:

Distance from one end :	0	3	6	9	12	15	18
Areas :	27.9	30.6	33.8	32.4	30.7	27.9	26.1

Find the volume of the solid.

Solution: Areas are considered as ordinates,

$y_1=27.9$, $y_2=30.6$, $y_3=33.8$, $y_4=32.4$, $y_5=30.7$ $y_6=27.9$, $y_7=26.1$; d=3

According to Simpsons one third rule,

Required area = $[(y_1+y_7)$ $+2(y_3+y_5)$ $+4(y_2+y_4+y_6)]$

= [(27.9 + 26.1) + 2(33.8 + 30.7) + 4(30.6 + 32.4 +27.9)] cub.unit

= [54 + 129 + 363.6] cub unit = 546.6 cub. unit .

VOLUME AND SURFACE AREAS OF REGULAR SOLIDS:

we had discussed about plane figures. Here we will discuss about solids. Solids are three dimensional figures. Here, we will obtain the volume and surface areas of the following solids: Cuboids, Prism, Cylinder, Sphere, Cone and Pyramid.

3.3.1 **Cuboid:** A Cuboid is a solid bounded by six rectangular faces. It has three dimensions, length, breadth and height. The face on which the Cuboid rests is called its *base*, the four faces which meet the base are called *lateral faces* and the face opposite to the base is the *top.*

a Cuboid with length l, breadth b and height h.

.

(a) Volume : V= Area of the base x height
= lbh cub. unit

(b) Lateral Surface Area = Sum of the Area of the lateral faces
= 2(l +b).h sq. unit

(c) Total Surface Area = Lateral Surface Area +Area of Base and Top
= 2(lb +bh +lh) sq. unit

(d) Length of a diagonal = unit

(e) Total Length of the Edges = 4(l+b+h) unit

A *Cube* is a special case of a Cuboid. All the edges of a Cube are equal.

For the Cube ABCDEFGH:
length of an edge = a unit

(a) Volume : V = a^3 cub. unit

(b) Lateral Surface Area = $4a^2$ sq. unit

(c) Total Surface Area = $6a^2$ sq. unit

(d) Length of a diagonal = a unit

(e) Total Length of the Edges = 12a unit

Ex. 1. The dimensions of a cuboid are in the ratio 1:2:3 and its total surface area is 352 sq. m. Find the dimensions and also the volume of the Cube.

Solution: The dimensions are in the ratio 1:2:3. Let the dimensions be x , 2 x , and 3 x

Example 9 (2013) : The cross sectional area of a tunnel is as follows:
Distance from one end :
0 3 6 9 12 15 18

Areas : 27.9 30.6 33.8 32.4 30.7 27.9 26.1
Find the volume of the solid.
Solution: Areas are considered as ordinates,
$y_1=27.9$, $y_2=30.6$, $y_3=33.8$, $y_4=32.4$, $y_5=30.7$ $y_6=27.9$, $y_7=26.1$; $d=3$
According to Simpsons one third rule,
Required area = $[(y_1+y_7) +2(y_3+y_5) +4(y_2+y_4+y_6)]$
= [(27.9 + 26.1) + 2(33.8 + 30.7) + 4(30.6 + 32.4 +27.9)] cub.unit
= [54 + 129 + 363.6] cub unit = 546.6 cub. unit .

VOLUME AND SURFACE AREAS OF REGULAR SOLIDS:

we had discussed about plane figures. Here we will discuss about solids. Solids are three dimensional figures. Here, we will obtain the volume and surface areas of the following solids: Cuboids, Prism, Cylinder, Sphere, Cone and Pyramid.

3.3.1 **Cuboid:** A Cuboid is a solid bounded by six rectangular faces. It has three dimensions, length, breadth and height. The face on which the Cuboid rests is called its *base*, the four faces which meet the base are called *lateral faces* and the face opposite to the base is the *top.*

a Cuboid with length l, breadth b and height h.

.

(a) Volume : V= Area of the base x height
= lbh cub. unit
(b) Lateral Surface Area = Sum of the Area of the lateral faces
= 2(l +b).h sq. unit
(c) Total Surface Area = Lateral Surface Area +Area of Base and Top
= 2(lb +bh +lh) sq. unit
(d) Length of a diagonal = unit
(e) Total Length of the Edges = 4(l+b+h) unit

A *Cube* is a special case of a Cuboid. All the edges of a Cube are equal.

For the Cube ABCDEFGH:
length of an edge = a unit

(a) Volume : V = a^3 cub. unit
(b) Lateral Surface Area = $4a^2$ sq. unit
(c) Total Surface Area = $6a^2$ sq. unit
(d) Length of a diagonal = a unit
(e) Total Length of the Edges = 12a unit

Ex. 1. The dimensions of a cuboid are in the ratio 1:2:3 and its total surface area is 352 sq. m. Find the dimensions and also the volume of the Cube.

Solution: The dimensions are in the ratio 1:2:3. Let the dimensions be x , 2 x , and 3 x

Example 9 (2013) : The cross sectional area of a tunnel is as follows:

Distance from one end :

0 3 6 9 12 15 18

Areas : 27.9 30.6 33.8 32.4 30.7 27.9 26.1

Find the volume of the solid.

Solution: Areas are considered as ordinates,

$y_1=27.9$, $y_2=30.6$, $y_3=33.8$, $y_4=32.4$, $y_5=30.7$ $y_6=27.9$, $y_7=26.1$; d=3

According to Simpsons one third rule,

Required area = $[(y_1+y_7) +2(y_3+y_5) +4(y_2+y_4+y_6)]$

= [(27.9 + 26.1) + 2(33.8 + 30.7) + 4(30.6 + 32.4 +27.9)] cub.unit

= [54 + 129 + 363.6] cub unit = 546.6 cub. unit .

VOLUME AND SURFACE AREAS OF REGULAR SOLIDS:

we had discussed about plane figures. Here we will discuss about solids. Solids are three dimensional figures. Here, we will obtain the volume and surface areas of the following solids: Cuboids, Prism, Cylinder, Sphere, Cone and Pyramid.

3.3.1 **Cuboid:** A Cuboid is a solid bounded by six rectangular faces. It has three dimensions, length, breadth and height. The face on which the Cuboid rests is called its *base*, the four faces which meet the base are called *lateral faces* and the face opposite to the base is the *top.*

a Cuboid with length l, breadth b and height h.

.

(a) Volume : V= Area of the base x height

= lbh cub. unit

(b) Lateral Surface Area = Sum of the Area of the lateral faces

= 2(l +b).h sq. unit

(c) Total Surface Area = Lateral Surface Area +Area of Base and Top

= 2(lb +bh +lh) sq. unit

(d) Length of a diagonal = unit

(e) Total Length of the Edges = 4(l+b+h) unit

A *Cube* is a special case of a Cuboid. All the edges of a Cube are equal.

For the Cube ABCDEFGH:

length of an edge = a unit

(a) Volume : V = a^3 cub. unit

(b) Lateral Surface Area = $4a^2$ sq. unit

(c) Total Surface Area = $6a^2$ sq. unit

(d) Length of a diagonal = a unit

(e) Total Length of the Edges = 12a unit

Ex. 1. The dimensions of a cuboid are in the ratio 1:2:3 and its total surface area is 352 sq. m. Find the dimensions and also the volume of the Cube.

Example 9 (2013) : The cross sectional area of a tunnel is as follows:
Distance from one end : 0 3 6 9 12 15 18

Areas : 27.9 30.6 33.8 32.4 30.7 27.9 26.1
Find the volume of the solid.
Solution: Areas are considered as ordinates,
$y_1=27.9$, $y_2=30.6$, $y_3=33.8$, $y_4=32.4$, $y_5=30.7$ $y_6=27.9$, $y_7=26.1$; $d=3$
According to Simpsons one third rule,
Required area = $[(y_1+y_7)$ $+2(y_3+y_5)$ $+4(y_2+y_4+y_6)]$
= [(27.9 + 26.1) + 2(33.8 + 30.7) + 4(30.6 + 32.4 +27.9)] cub.unit
= [54 + 129 + 363.6] cub unit = 546.6 cub. unit .

VOLUME AND SURFACE AREAS OF REGULAR SOLIDS:

we had discussed about plane figures. Here we will discuss about solids. Solids are three dimensional figures. Here, we will obtain the volume and surface areas of the following solids: Cuboids, Prism, Cylinder, Sphere, Cone and Pyramid.

3.3.1 **Cuboid:** A Cuboid is a solid bounded by six rectangular faces. It has three dimensions, length, breadth and height. The face on which the Cuboid rests is called its *base*, the four faces which meet the base are called *lateral faces* and the face opposite to the base is the *top.*

a Cuboid with length l, breadth b and height h.

.

(a) Volume : V= Area of the base x height
= lbh cub. unit
(b) Lateral Surface Area = Sum of the Area of the lateral faces
= 2(l +b).h sq. unit
(c) Total Surface Area = Lateral Surface Area +Area of Base and Top
= 2(lb +bh +lh) sq. unit
(d) Length of a diagonal = unit
(e) Total Length of the Edges = 4(l+b+h) unit

A *Cube* is a special case of a Cuboid. All the edges of a Cube are equal.

For the Cube ABCDEFGH:
length of an edge = a unit

(a) Volume : V = a^3 cub. unit
(b) Lateral Surface Area = $4a^2$ sq. unit
(c) Total Surface Area = $6a^2$ sq. unit
(d) Length of a diagonal = a unit
(e) Total Length of the Edges = 12a unit

Ex. 1. The dimensions of a cuboid are in the ratio 1:2:3 and its total surface area is 352 sq. m. Find the dimensions and also the volume of the Cube.

Example 9 (2013) : The cross sectional area of a tunnel is as follows:

Distance from one end :	0	3	6	9	12	15	18
Areas :	27.9	30.6	33.8	32.4	30.7	27.9	26.1

Find the volume of the solid.

Solution: Areas are considered as ordinates,

$y_1=27.9$, $y_2=30.6$, $y_3=33.8$, $y_4=32.4$, $y_5=30.7$ $y_6=27.9$, $y_7=26.1$; $d=3$

According to Simpsons one third rule,

Required area = $[(y_1+y_7) +2(y_3+y_5) +4(y_2+y_4+y_6)]$

= [(27.9 + 26.1) + 2(33.8 + 30.7) + 4(30.6 + 32.4 +27.9)] cub.unit

= [54 + 129 + 363.6] cub unit = 546.6 cub. unit .

VOLUME AND SURFACE AREAS OF REGULAR SOLIDS:

we had discussed about plane figures. Here we will discuss about solids. Solids are three dimensional figures. Here, we will obtain the volume and surface areas of the following solids: Cuboids, Prism, Cylinder, Sphere, Cone and Pyramid.

3.3.1 **Cuboid:** A Cuboid is a solid bounded by six rectangular faces. It has three dimensions, length, breadth and height. The face on which the Cuboid rests is called its *base*, the four faces which meet the base are called *lateral faces* and the face opposite to the base is the *top.*

a Cuboid with length l, breadth b and height h.

.

(a) Volume : V= Area of the base x height
= lbh cub. unit

(b) Lateral Surface Area = Sum of the Area of the lateral faces
= 2(l +b).h sq. unit

(c) Total Surface Area = Lateral Surface Area +Area of Base and Top
= 2(lb +bh +lh) sq. unit

(d) Length of a diagonal = unit

(e) Total Length of the Edges = 4(l+b+h) unit

A *Cube* is a special case of a Cuboid. All the edges of a Cube are equal.

For the Cube ABCDEFGH:
length of an edge = a unit

(a) Volume : V = a^3 cub. unit

(b) Lateral Surface Area = $4a^2$ sq. unit

(c) Total Surface Area = $6a^2$ sq. unit

(d) Length of a diagonal = a unit

(e) Total Length of the Edges = 12a unit

Ex. 1. The dimensions of a cuboid are in the ratio 1:2:3 and its total surface area is 352 sq. m. Find the dimensions and also the volume of the Cube.

Example 9 (2013) : The cross sectional area of a tunnel is as follows:

Distance from one end : 0 3 6 9 12 15 18

Areas : 27.9 30.6 33.8 32.4 30.7 27.9 26.1

Find the volume of the solid.

Solution: Areas are considered as ordinates,

y_1=27.9, y_2=30.6, y_3=33.8, y_4=32.4, y_5=30.7 y_6=27.9, y_7=26.1 ; d=3

According to Simpsons one third rule,

Required area = $[(y_1+y_7) +2(y_3+y_5) +4(y_2+y_4+y_6)]$

= [(27.9 + 26.1) + 2(33.8 + 30.7) + 4(30.6 + 32.4 +27.9)] cub.unit

= [54 + 129 + 363.6] cub unit = 546.6 cub. unit .

VOLUME AND SURFACE AREAS OF REGULAR SOLIDS:

we had discussed about plane figures. Here we will discuss about solids. Solids are three dimensional figures. Here, we will obtain the volume and surface areas of the following solids: Cuboids, Prism, Cylinder, Sphere, Cone and Pyramid.

3.3.1 **Cuboid:** A Cuboid is a solid bounded by six rectangular faces. It has three dimensions, length, breadth and height. The face on which the Cuboid rests is called its *base*, the four faces which meet the base are called *lateral faces* and the face opposite to the base is the *top.*

a Cuboid with length l, breadth b and height h.

.

(a) Volume : V= Area of the base x height
= lbh cub. unit

(b) Lateral Surface Area = Sum of the Area of the lateral faces
= 2(l +b).h sq. unit

(c) Total Surface Area = Lateral Surface Area +Area of Base and Top
= 2(lb +bh +lh) sq. unit

(d) Length of a diagonal = unit

(e) Total Length of the Edges = 4(l+b+h) unit

A *Cube* is a special case of a Cuboid. All the edges of a Cube are equal.

For the Cube ABCDEFGH:
length of an edge = a unit

(a) Volume : V = a^3 cub. unit

(b) Lateral Surface Area = $4a^2$ sq. unit

(c) Total Surface Area = $6a^2$ sq. unit

(d) Length of a diagonal = a unit

(e) Total Length of the Edges = 12a unit

Ex. 1. The dimensions of a cuboid are in the ratio 1:2:3 and its total surface area is 352 sq. m. Find the dimensions and also the volume of the Cube.